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Mathematical Modeling in Problem Situations of Daily Life

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Abstract

The study focuses on the elements involved in mathematical modeling. Some of the problem situations dealt with include the following: athletics, throwing a ball, the time it takes to warm up a car or an oven, filling and emptying containers, cycling, the movement of a wheel as it turns upon its axis or that turns without skidding, a pendulum, a real and a toy car, the movement of a motorcycle. The video that was taped by the students was treated using the Tracker software and analyzed in order to relate the verbal, graphic, analytical and numerical approaches with the problem situation chosen by each collaborative group. We have concluded that this manner of working generates knowledge, interest in learning mathematics individually and collaboratively, as well as fostering values the likes of punctuality, honesty, responsibility and respect, to mention only a few, all of which are so necessary in current society.

Keywords: mathematical modeling, problem situation, collaborative work, Tracker, semiotic representations.

1. Introduction

Mathematics has been used in life ever since the very existence of human beings. Different civilizations throughout time have left a cumulus of mathematics knowledge of varying types and in different contexts. Each of said portions of knowledge was in keeping with the needs that prevailed at different points in history and had the objective of providing answers to all phenomena, be they from the realm of physics, chemistry, astronomy, music, astrology, art or religion. It would seem, however, that at a certain point mathematics teachers forgot to relate such problems of daily life with the mathematics raised in classrooms which, according to Arrieta, *et al,* (2007) and Suárez (2014), makes their teaching and learning more interesting.

In the classroom setting, an algorithmic mathematics is taught that does not relate to teacher or student contexts. At times that mathematics is completely out of place, with no specific function within the applications of some area of knowledge, standing just within mathematics itself.

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For instance, in an integration exercise pertaining to a Calculus course, students are asked to calculate the integral $\int_0^1 \sqrt{1+x}dx.$ In order to do this, the variable is changed t=1+x, dt=dx, the new limits of integration are calculated, if $x=0 \Rightarrow t=1$ and $x=1 \Rightarrow t=2$, and one moves on to calculating the integral $\int_0^1 \sqrt{1+x}\,dx = \frac{2}{3}\left(\sqrt{8}-1\right).$ Another option for calculating the integral is to use software that immediately provides the result: $\int_0^1 \sqrt{1+x}dx = 1.21895142$.

In another context, students are asked to calculate the arc length of the function $f(x) = \frac{2}{3}x^{\frac{3}{2}}$ of x=0 a x=1, for which one substitutes the first derivative, $\frac{dy}{dx} = x^{\frac{1}{2}}$ in the formula to calculate the arc length and arrive at the same integral.

Another approach is to provide the graph (See Figure 1) and one asks that the arc length be calculated from point (0, 0) to point (1, 2/3). In this case the problem leaves the context of traditional teaching because students are only given the chart without the analytical expression for applying the formula in order to calculate the arc length.

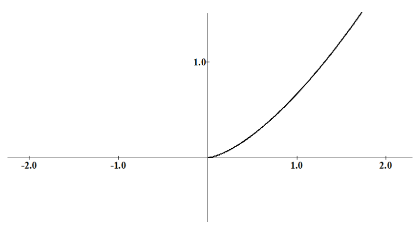


Figure 1: Graph of the function

The teacher can use several approaches for the same exercise in his/her teaching in the classroom, however in this case there is no connection between mathematics and its setting, which makes students uneasy and encourages them to question their teachers, asking things like: What do mathematics do for me? In what situations of daily life are they useful? Those types of questions are derived from the fact that traditional teaching is deeply rooted in educational institutions, disconnected from their context. They are focused on observing development of an exercise on the blackboard, to subsequently apply it under similar conditions, like imitating the teacher's work.

The study reported here raises the didactic strategy in which students build mathematical modeling competence, based on development of activities within different problem situations (Hitt, 2007, 2013) related to their contexts and supported by the theory of semiotic representations, visualization, problem solving, collaborative work and the Tracker software. The Tracker plays a predominant role in the strategy because as of the video taping of a problem situation, students interpret and relate the data, graphs and the functions provided by the software as the activity is developed, in which the modeling action becomes clear (Ezquerra, Iturrioz & Díaz, 2012; Pantoja, Ulloa & Nesterova, 2013). We agree with Cordero & Suárez (2005) in that it is important to the mathematics teaching and learning process for school practice to include circumstances, phenomena, successes, etc., that foster appearance of mathematics in the daily lives of students within contexts that differ from those dealt with in the traditional classroom (Arrieta & Díaz, 2015; Hitt & Cortés, 2009; Arrieta, *et al*, 2007; Ezquerra, s/f, 2005, 2010). Examples of these contexts include athletics, throwing a ball, the time it takes to warm up a car or an oven, filling and emptying containers, cycling, the movement of a wheel as it turns upon its axis or that turns without skidding, a spring or a pendulum, the movement of a real and a toy car, the movement of a motorcycle or photographs of water squirting in a fountain, or the arch of a church or of a rainbow on a rainy sunny day.

With development of the problem situations cited, in which students play the leading role, the intention is for them to obtain real time data and charts from the Tracker program based on the video. They then analyze and interpret the data and the charts, as opposed to what takes place in the classroom, for instance, where there is a set of data extracted from textbooks or fictitiously invented by the teacher.

From analysis of the videotapes, the survey, the interview, the presentation and the reports, we argue that mathematical modeling fosters a positive effect among students with respet to their motivation to learn mathematics and the discipline's articulation with problem situations within the everyday life setting. This is because not only were they able to adjust the polynomials, but they were also able to relate and interpret the graphs and data to movement of the objects in real time. On the qualitative side, the students showed that they were interested and motivated by this new way of learning mathematics; they were punctual, respectful and participatory.

1. Mathematical Modeling in the School Setting

Researchers in mathematics education are said to be very concerned about having students learn significantly. One of the recommendations to achieve said objective is generation of didactic activities that relate the context in which students develop their daily lives with mathematics. The purpose of doing so is to promote, among students, discovery, exploration, intuition and motivation to learn mathematics.

In recent years the strategies used to learn mathematics from real world problem situations (Blum, 2004) have gained strength given that they facilitate interpretation of reality based on detection of participating variables and collection of data generated for modeling those situations. That is to say, linking daily life to mathematics.

Mathematical modeling is one of the options available to teachers in order to unleash student motivation to learn mathematics. In addition to building a mathematical model of a problem situation, it promotes thought concerning the behavior of the variables involved and how they tie in to the phenomenon.

In particular terms, modeling a problem situation is a mathematical representation of an object and it includes signs or figures that act as mathematical expressions of the concept. Modeling (See Figure 2) in the school arena is understood to be a practice (of reference) exercised by teachers and students in a certain context and time in response to a situation or phenomenon of the external world, albeit close to student reality, individually and collectively, by way of the process of interaction (Córdoba, 2011, pg. 10).

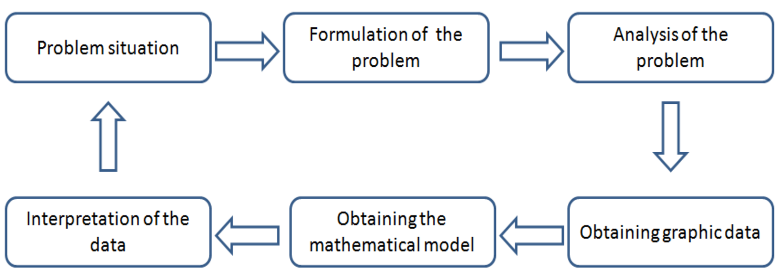


Figure 2. **Flowchart of the process of mathematical modeling.**

According to Blomhoj (2009), modeling activities make the learning process possible and enable establishment of cognitive roots upon which mathematical concepts are built. Hence mathematical modeling (Arrieta and Díaz, 2015) has the purpose of describing and analyzing situations from daily life in order to motivate learning of mathematics.

For Freundental (1980, *pg. 20*), the ideal teaching sequence is contrary to what generally happens in mathematics classrooms. That is to say, much time is devoted to carrying out calculations based on repeating exercises, but very little or none on undertaking practices with problems in real contexts. Therein lays the importance of re-broaching the sense raised by Freundenthal with respect to mathematical modeling. Undoubtedly the most transcendental result is that use of real situations motivates students to learn mathematics, given that they show interest during the process. It moreover facilitates retention of everything that can possibly be built and that makes sense in their context, as well as the collaborative coexistence that fosters sharing ideas, participation, respect, honesty and punctuality, amongst other values that are so very necessary in today's Mexican society.

2. Validation of the real data and the Tracker data

Study of the trajectory of a cannonball shot from a cannon is currently classified as parabolic motion. However in the 16th Century, people thought that the projectile's motion consisted of three trajectories (See Figure 3): one in a straight line, one curved and another in a straight line headed toward the center of the Earth (Tartaglia, 1537 cited by Martínez & Guevara, 1998). The process carried out by Tartaglia is a clear example of the fact that mathematics are useful and applicable in practical and real situations; that they enable conjectures, predictions and assumptions that can later be used in new situations to find answers and solutions to problems of interest to society; but that they are also sources that generate mistakes, perhaps involuntary, albeit still considered a part of building knowledge.

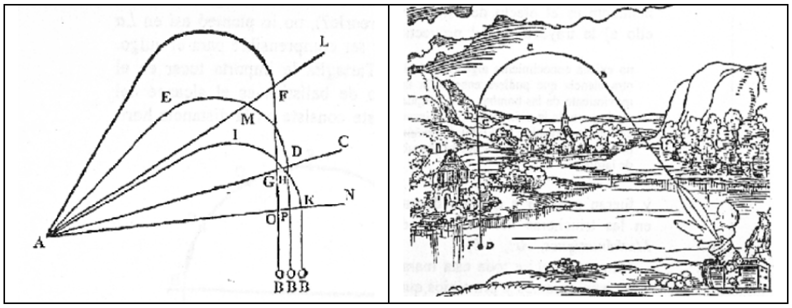


Figure 3: **Figures that allude to the motion of projectiles.**

The foregoing type of erroneous conception of projectile motion is not accidental. It is part of the problem generated when a person faces an unknown situation that he/she does not know how to argue. But the conception is still considered part of the learning process, in this case of mathematical modeling. In other words, it is a dialectic process that students, in their search for a solution, go through until they achieve or not the desired result.

Galileo studies in depth the motion of projectiles and in the *Discorsi*, he raises the following assertion: *"A projectile, while it moves by a motion compounded of a uniform horizontal motion* (graph *x* vs. *t*) *and a motion naturally accelerated downward* (graph *y* vs. *t*)*, describes under this movement a semi-parabolic line."* G. Galilei (1633, pg. 129).

The projectile motion studied by Galileo, in our case throwing a ball toward a basketball hoop, was analyzed using the Tracker and used to validate the information shown on the computer screen. This was because one of the situations that concerned the students was whether the data and charts obtained from the video and shown by the software corresponded to the real measurements of the scene filmed.

It was assumed that the students know that motion consists of an object's change of position with respect to time. It was also important to adequately locate the system of reference and a unit of measure in the Tracker, as it performs the function of interface between the problem situation (real) and the digital video (virtual) using the software for manipulation. Consequently, modeling a ball being thrown can be a non-fictitious approach that differs from the exercises raised in a tradictional Physics textbook that refer to the motion of a projectile. In the latter, conventional data are proposed for some of the variables included, which are substituted in the motion equations and the solution is determined by way of algebraic manipulation, without considering geomnetric or numerical approaches.

In the case of the proposal, a student is placed on a basketball court and the trajectory of the ball heading toward the hoop is videotaped. The Tracker processes the digital video, and "gives away" the charts along the planes of ***t-x***, ***t-y***, ***x-y***, a table containing numerical data of the variables involved and the polynomials of the motion along the respective planes (See Table 1). This analysis of the parabolic throw does not contradict the tradicitonal proposals of Physics textbooks. It rather supplements those proposals and enriches student learning because with ICTs students are given the opportunity –from preparing the videotaping set right through to the group discussion session- to relate the projectile's motion to the interpretation of the charts, data and quadratic equations provided by the Tracker.

Table 1: ***Image of the throw; charts and equations of the ball throw.***

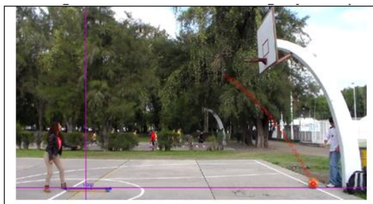
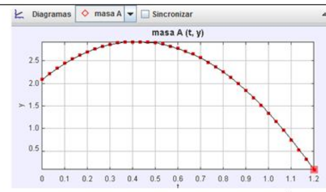
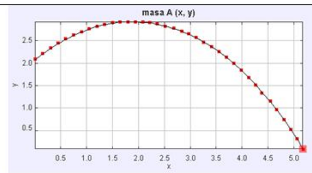
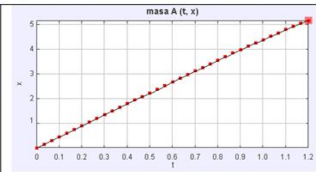
	 $y = 2.009 + 3.956t - 4.7t^2$		(1)
 $y = 2.053 + .9620x - .2586x^2$	(2)	 $x = .04452 + 4.342t$	(3)

Table 2 presents the equations for the parabolic throwing motion (Resnick, Halliday & Krane, 2008) that are used to obtain the remaining data of the parabolic motion. In this case, the acceleration of gravity, the angle of the throw and the initial velocity, with a comparison among equations (1) & (5), (2) & (10), (3) & (4) term to term and by way of algebraic manipulation, approximate values for the acceleration of gravity $g = 9.4\,m/s^2$, initial velocity $v_0 == 5.874\,m/s$ and the angle of the throw $\theta = 42.334^\circ$ are determined. It should be noted that the data produced for acceleration of gravity is close to the value $g = 9.81\frac{m}{s^2}$, which given the circumstances of context, such as the inexperience of teacher and students in videotaping, the weather conditions or handling the Tracker program, one can deem the data to be a good approximation that was put to use so that the students could discuss the reason the error was generated.

Table 2*:* ***Motion Equations.***

$x = x_0 + v_0\cos(\theta)t$	(4)	$y = y_0 + v_0 sen(\theta)t - \frac{1}{2}gt^2$	(5)	$v_x = v_0\cos(\theta)$	(6)	$a_x = 0$	(7)
$v_y = v_0 + v_0 sen(\theta) - gt$	(8)	$a_y = -g$	(9)	$y = y_0 + \tan(\theta)x - \frac{g}{2v_0^2\cos^2(\theta)}x^2$	(10)		

This treatment is considered a validation of the media and materials used. Those media and materials should be valued given that at times the interface between the real situation and the Tracker-processed video produces circumstances that lead the agents to doubt the process that deals with the relationship exisitng between mathematics and the real world.

3. Digital Video on Teaching Mathematics

As Jofrey states (2005, 2010), introduction of digital video in the classroom setting has been stronger force than analogue video because VCR controls encumber the task of consulting the video. Taping devices and programs used to manipulate videos are different. And this is one of the reasons that the advisability of using digital video in the classroom has been demonstrated over the past few years, as it enables achieving learning objectives based on the expression and communications potential that video offers. One of the benefits of video analysis technology is that students can visualize several representations of the same problem situation. For instance, from the video of the motion of a motorcycle, students can visualize a picture photograph, tables of data, graphs, mathematical formulae and verbal and written descriptions (Joffrey, 2005).

At present we live in a society that is increasingly visual in nature; students are increasingly interested in consulting videos of their interest in databases on the Internet, added to the reduced cost of video cameras and technological developments that facilitate use and distribution of digital educational media and materials.

Production of educational video clips supports teaching because it offers the opportunity to understand and develop intellectual activities during the process, and promote having the students themselves become the creators or designers so as to achieve greater depth in the study themes.

Video tapes of moving objects or of real situations facilitate for teachers the task of incorporating into the classroom authentic research that allow the students, with the help of specialized software, to improve their comprehension of the concepts to be learned, as well as to create graphic, analytical and numerical representations of problem situations that relate to daily life. This is pointed out by Calderón, Núñez & Gil (2009), where they use a digital camera as a Physics lab tool to study a projectile thrown by a home-made device, and where the objective was to compare the theoretical predictions against experimental results.

Using the Tracker, the students can “mark” the position of an object in each of the frames of a video clip so as to obtain information concerning the position and velocity, indicating the frames per second (FPS) of video (See Figure 3), choose the section of video that is of interest and calibrate the interface in order to relate the measures of the real scenario with the computer screen, amongst many other functions that can be undertaken using the program’s routines.

4. The Theory of Raymond Duval

It is known that traditional lectures aim for the algorithmic, leaving aside what Duval (1995) calls the semiotic representations of a mathematics object, a sign system that makes it possible to perform functions of communication, treatment and objectivation. In the study reported here, the system of semiotic representation was useful because the teacher raised orally and in writing different problem situations related to their contexts so that the students could choose the problem they wanted to work with, to later show the analytical, numerical and graphic records. In the case of throwing the ball, the problem related to the Parabola mathematical object and its semiotic representations, namely:

- the graphs of the parabola in its different positions on the Cartesian plane;
- the problems of daily life; in this case, throwing the ball toward the basketball hoop;
- the equations, as a function of their position on the Cartesian plane;
- the ordered pairs, obtained from a measurement of the variables in a problem situation; and
- the description of the mathematical object in common language.

None of said semiotic representations are obtained naturally (Figure 5) since it is the teacher who must design activities so that the students are able to appropriate them, that is to say, pursuant to communicative purposes.

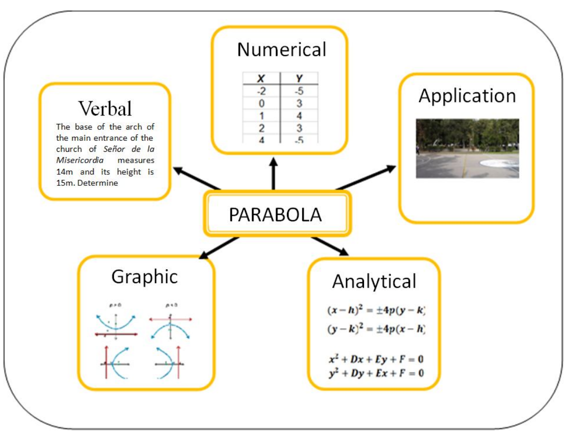


Figure 5: Semiotic records of the parabola object.

For Duval (2006), intellectual activity essentially consists of transforming semiotic representations, which are of two types, namely: treatment and conversion. Treatment happens when a transformation produces another within the same register and this makes the choice of the “better” change of register relevant for resolving a given problem, by internally transforming the register.

In the Parabola mathematical object, different types of transformations within the same register are done. For instance, if one considers the analytical register, the treatment can be applied to the regular equation of the parabola $(x+2)^2 = -4(y+1)$ to obtain the general equation, or vice versa. The conversion, according to Duval (1995), refers to transformation of the representation into another representation of another register in which all or part of the meaning of the initial representation is maintained; it is the change from one register to another. Hence, for instance, from the graphic register to the analytical register (Figure 6) or making the conversion from the verbal register to the graphic, numerical or analytical register so that students can interiorize their knowledge.

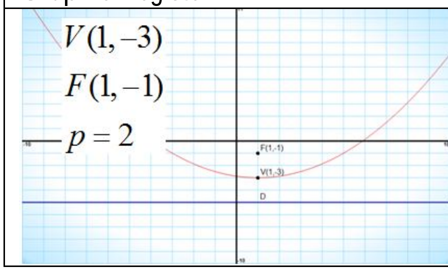
Graphic Register	Analytical Register
	$(x-h)^2 = \pm 4p(y-k)$ $(x-1)^2 = 4(2)(y-k)$ $x^2 - 2x - 8y - 23 = 0$

Figure 6: Conversion of the graphic register to the analytical register.

Quick and spontaneous conversion of coordination of at least two representation is the basis of the comprehension (integral) of a conceptual content, a situation that a student had to face when the computer screen showed the video of the ball in motion, three graphs that represent the motion and the numerical data of position with respect to time and its position. With another routine, students are presented with the option of choosing the function that best suits the data; in short, the students working collaboratively had to interpret the different conversions between the register of the parabola object with the problem situation of the projectile's motion.

5. Information and Communications Technology (ICT)

Today's society is a daily user of digital technology. Suffice it to see that the majority of people have a mobile phone, iPod, laptop, calculator, grapher, electronic tablet and electronic agenda. This then begs the question: If society has positively valued use of digital technology in different social mediums, such as at home, work, services, inter alia, why has it not done so in the mathematics classroom as well? During the meetings of academics of Numerical Analysis of the CUCEI Department of Mathematics, discussions have been held on use of digital technology in the classroom and one of the permanent discussions deals with the reason teachers refuse to use such technology. Some of the arguments expressed are that use of digital technology brings with it the loss of arithmetic, geometric or thought skills among students, while teachers are not assiduous in working with such technology; they do not feel capable; they fail to stay up to date; and do not want to leave their comfort zones because doing so would imply work, time and dedication. As such, it is easier to simply not use ICTs in their classes.

It has been observed that students enrolled in the Numerical Analysis subject have a hard time observing and transferring the knowledge acquired from solution methods for non-linear equations, systems of linear equations, ordinary differential equations with initial values, interpolation and adjusting functions to problem situations within context. Consequently, we propose to integrate mathematical modeling so that students can work with a problem situation from everyday life, one in which it is interesting for them to seek the mathematical expression, with the support of ICTs that describe it, and then come back to explain the situation, as did the students in the plenary session.

6. ACODESA

In the ACODESA methodology, individual and collaborative group work is taken into consideration, as are in-class debates and self-reflection. ACODESA is an adaptation of a social interactionism approach to learning mathematics (Hitt, 2007; Hitt *et al.*, 2008; Gonzalez-Martin *et al.*, 2008, Hitt & Cortés, 2009; Hitt & González-Martin, 2015; Páez, 2004). In the first three phases, the teacher acts as guide and the students are allowed to argue and validate their productions in the institutionalization process (See Table 3).

Table 3: ***ACODESA methodology and its connection to the study proposed.***

Individual work (production of functional representations to understand the problem situation).	Takes place in student participation in the course-workshop, in modeling the problem situation and in designing the videotaping scenario, for example when throwing the ball or running a race.
Teamwork on one and the same situation. Discussion and validation process (fine tuning the functional representations).	Presented in the tape of selected problem situations, in procurance of the mathematical model and in drafting of the work reports that were prepared.
Debate (that can become a scientific debate).	On the process of procuring and interpreting the mathematical model, presentation of the work, drafting the reports and conclusions.
Individual work: reconstruction and self-reflection.	During the final phase, the students were asked to do the process with a problem situation, and to undertake the corresponding analysis.
Institutionalization.	Takes place during presentation of the collaborative group work, during the group discussion and when revising the reports handed in.

7. Results

8.1 Description of the Field Work Phase

The following are the problem situations dealt with: motion, runner, cyclist, real and toy automobiles, motorcyclist and skater; throwing a ball in the games of volleyball, basketball, soccer, American football, jai alai (or fronton) and free-fall; turning of a bicycle wheel without skidding and turning upon its own axis; filling and emptying containers; movement of a yo-yo. The sets for videotaping were the sports unit and the Olympic Coliseum at the CUCEI, the classroom and a local highway. Before taping all scenes, we spoke with the students of the importance of relating the mathematical concepts learned in the classroom in everyday situations. We also made it clear that the video camera had to be placed in a perpendicular position with respect to the set, that the unit of measurement had to be visible and that the actor or object in motion had to have an emblem that was at all times visible in the video, as this generates less data error during the transition from real to virtual. Figure 7 shows the videotaping sets for free-fall and basketball. The students had fun. They showed themselves to be interested and happy because they were having fun in their mathematics class; they had not experienced this very often -gathered from the comments they made to each other and extended to the teacher as well. We also observed that not many of the students are accustomed to playing; they did not throw the ball well or at the beginning it took some time before they participated; at the end all were agreeable and it was not a hardship to play and learn mathematics.

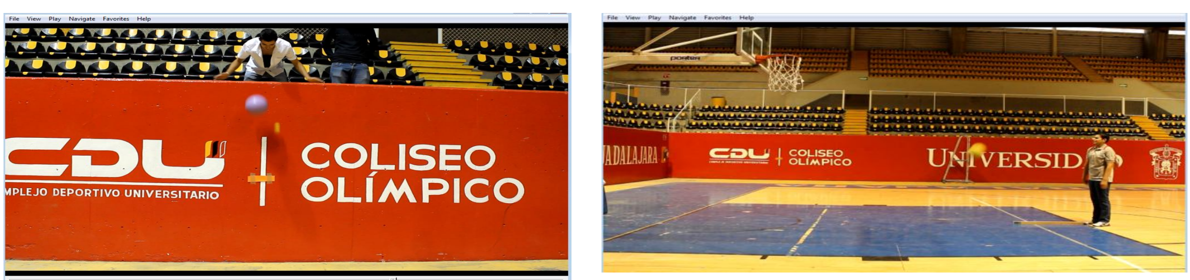


Figure 7: **Images of dropping an object and throwing a ball towards the basketball hoop.**

Below are discussions that were fostered between the teacher and students concerning three problem situations.

8.2 Free-fall

It is interesting how the fieldwork drew the students' attention. For instance, the dialogue between teacher and students in the case of the free-fall (Table 4) shows that the collaborative work promoted the discussion concerning what the most advisable position for the axis was, the cut of the video on motion of the object in question and the movement that would be analyzed, the number of frames that would be taken into account, amongst other parameters.

Table 4. *Conversation between 4 students and the teacher concerning the free-fall problem situation.*

Pedro: Teacher…in this video (free-fall of the ball), the wind moves it a bit off the x axis. Should we tape that? Should we take it as though it were a margin

Teacher: Do you think it could cause a problem when it (the ball) goes off track?

Thelma: Well instead… what we're interested in is the free-fall… in other words "y".

Pedro: What we're proposing or what we're interested in is the fall at "y" … so … there is a margin of error if we take the x because it will go off track … but that is due more to external causes because of the ball's material or the presence of wind … so what we did … hmmm … well the "y" … the question is … for example … here (points to the Tracker video screen) we finish the cut (of the video) when the ball hits the ground or do we include it's (the ball's) bounce too?...

A member of another group interrupts the conversation and answers the following:
Just the free-fall.

Teacher: You can tape the entire movement and do the adjustment for it all …from the time you let it (the ball) go until it bounces …or even after it has bounced out of the frame and tape the entire movement … but … you will all decide what interests you … you can do the entire treatment on Tracker … you are free to locate it …you know, from here to here (makes hand gestures indicating the free-fall of the ball) this is what interests me and I won't take the rest into account …whatever you decide is right. Let's see …if you tape until the bottom (referring to the fall of the ball until it hits the ground) is it still free-fall?

Luis Fernando: Yes.

Teacher: Ah, then you can decide that from here to here is what interests you and that will be what you'll work with.

Luis Fernando: So it can be until the measurement of the ruler?

Thelma: Yes

Teacher: Yes, you can tape it that way.

Thelma: And our axis, does it matter if we put it high or low?

Luis Fernando: It would just be …changing the sign, right? If it's high (referring to the location of the axis when the ball is dropped).

Teacher: Exactly …either way, you'd still get the adjustment …the point where you want to take the reference point …the origin …if you want it on the floor or if it will be as of where the ball can be seen … like you say, it will just be the sign … if you locate it above, it will start at less.

Pedro: Ah…ok. Thanks.

8.3 The Cyclist

Another team analyzed the video of the cyclist's movement and in their conclusions (Table 5) one can see that their contributions attest to the fact that they understood and analyzed the motion; they thought about the factors involved, such as the duration of the video, the characteristics of the bicycle, and they were able to imagine and make conjectures in that respect. Moreover, based on the analysis undertaken regarding the cyclist, the students extrapolated several ideas, as they thought about everyday situations and situations that interested them, such as how to idealize the best soccer player in the world. The also related mathematics, modeling and analysis of real situations to a movie they had seen at some point in their lives (See Figure 8).

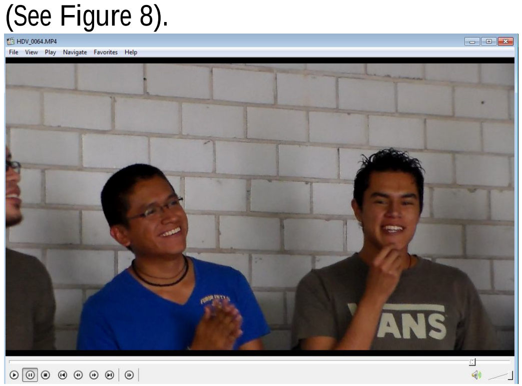


Figure 8. Members of group 3 during their presentation of the problem situation.

Table 5. *Conversation amongst students and teacher about the problem situation of the cyclist.*

Samuel: … As we could see from the video, the phenomenon is not constant …instead it has several variables that are not taken into account, that can at times generate certain noise when it is time to analyze them in a program that detects the phenomenon frame by frame. **Teacher:** Like which ones? **Samuel:** For example, the variable of pedaling down and that showed the point at which a parabolic motion arose and then a linear motion and afterwards again, in a way, it took on the performance it had shown before the foot descended together with the pedal. **Teacher:** Do you think that another factor, other than the one you've mentioned, would have been involved if the length of the video duration had been longer? **Samuel:** I don't think so because we have the theory that if the videotape had kept going, there would have been a point at which the cyclist would have maintained a constant speed and the graph would have been completely linear. I think that at that point there would not have been as many interferences …hmm …as many unexpected variables. **Teacher:** But don't you think that the cyclist would have tired, because you don't know, but the bicycle was very hard and the cyclist had to make a major effort to pedal it. **Guillermo:** It could happen, the cyclist could get tired or not control the bike properly or not have adjusted the gears properly. **Johor:** Also, I mean I think, that that bicycle was for doing tricks and the cyclist can't make a bicycle of that type go so fast. It would have to be a bicycle for racing, which is lighter, and has smoother gears… gears would need to be tighter. **Teacher:** But… even with all of that… if it was up to us to solve all these kinds of factors… Do you think the mathematical model would have been the same?... If the situation is the same, then would you have also gotten a parabola? **Everyone:** Yes. **Ricardo**: If we'd solved all those problems, the cyclist not getting tired or anything, supposedly the acceleration would be constant and that would give us the graph of a parabola… As for my conclusions… I think it's very interesting to do this kind of work because it makes you consider physics aspects of daily life and you start thinking… I don't know… for example, a physicist that is really good with these kinds of studies starts to analyze for instance the shots taken in a soccer or basketball game and like… I don't know… it would be interesting to see a physicist becoming the best player in the world. (laughter). **Teacher:** We would win the World Cup in Brazil. **Everyone:** Yes. (Laughter). **Ricardo:** There is a movie about a girl that studied physics and applied them to figure skating and it turned out that she could perform a move that seemed impossible and she did it just by studying the physics of it. **Teacher:** Very good guys. Thank you.

The kind of activities that are proposed here as well as the comments, conclusions and arguments from each collaborative group, suggest that the students can go beyond the mere application of an algorithm and that there are aspects of interest to them that are related to operations and mathematical algorithms. This situation ought to motivate teachers to include activities that interest students instead of simply learning algorithms that have been stripped of reality and their context.

Adjustment of the polynomial using Math Cad for displacement of the runner

Three videos were taped for the runner problem situation scenario: the first involves the runner starting from rest and then being asked to increase his/her velocity until arriving at the finish line; the second involves the runner arriving on the set while running and then maintaining his/her speed; and the third has a runner starting from rest, running until arriving at the finish line and runs back to the starting point.

This case included male and female participants, thus due to their very nature, there were significant differences in terms of weight, height, age, strength and agility, etc. For this problem situation, two situations are analyzed, namely:

a) A discussion was held concerning displacements of the trajectory of a man and of a woman, and in Table 6 a segment of the dialogue between the teacher and the students is shown. Figure 9 depicts the graphs that represents the movements of a male runner (i) and of a female runner (ii), and they are quite similar, which is stated by student César. In this case, the data differ because the man is faster than the woman, albeit in other cases she is as fast as or faster than some of the men.

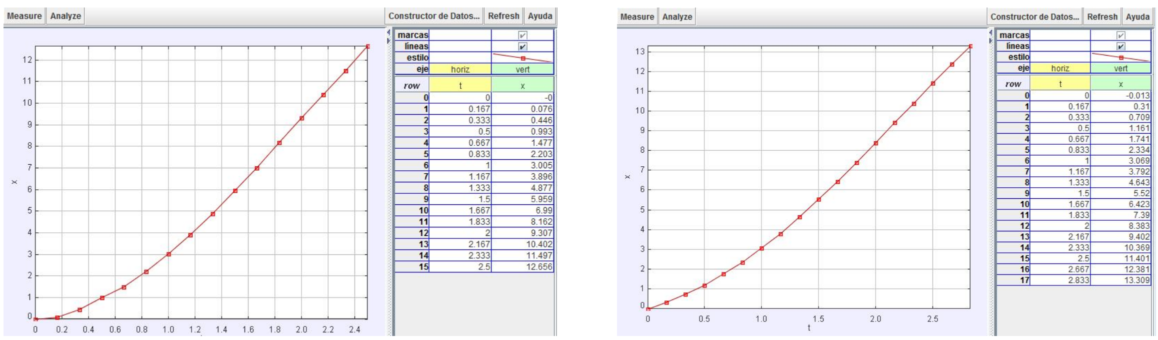


Figure 9: **Comparison of the graphs of one male and one female runner**

Table 6: Discussion of the problem situation of the runner

César: *Umm…well…I had a teacher in middle school who said that no one is just like you (pointing to a person), no one is just like you (pointing to someone else), I think that teacher never saw the universe from a mathematical perspective, if I showed him these 2 graphs* (see Figure 8) *and asked him (the teacher): What is the difference between these two graphs? … no, well… they both seem to be the same (presumed answer from the teacher)… and if I told him they are two different people.*

…This can tell us that we can obtain data and patterns from so much of what we observe on a daily, basis and by patterns I mean that any person… short, skinny, can have the same patterns, can have the same adjustment… with this same function we can analyze many people with this same movement, obviously…this one… not only running.

…I saw a movie once in which they analyzed the movement of sea buoys… for some war or something… I picture it somewhat like that… because normally the beach behaves a certain way… so when there are diversions from that behavior… if there is a ship, a submarine… a motorcycle or anything… well the buoy will have a different behavior… so then you can analyze it and obtain some kind of adjustment… well…get…whatever most frequently occurs and then any thing that occurs differently is understood to be something else, for instance a ship… you can know the size or the ship…umm… the mass of the ship…etc.…right?

…I think it's very interesting to see, well, … how one function can tell us about many different people…right? Because since we can recognize the same function as the fourth degree polynomial, we use it to analyze both runners… who themselves were very different, because one was a woman, a little bit shorter, she ran less; and the other was a man and ran faster; he was taller, slimmer and yet we can analyze both of them with the same function.

The reasoning and thinking of this student, shows that he related the mathematical modeling activity to real life situations, such as when he was in middle school and a film that was foreign to the situations that were worked on.

He also realized that certain mathematical functions can be used to apply to the movements of different people or different situations, as long as certain patterns are present and they are useful in different areas like oceanography, war or the movement of the runner.

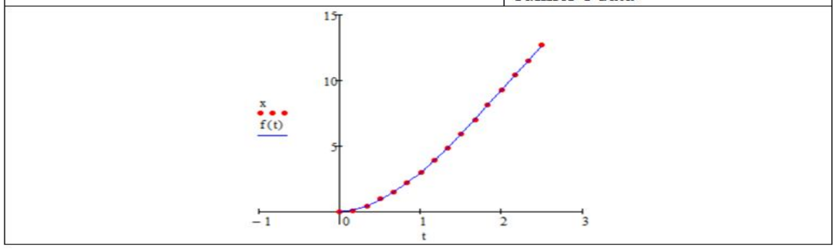
b) The students exported the data (see Table 7) calculated by the Tracker, based on treatment of the video, to the MatchCad program because the degree of the polynomial included in the Tracker's *"Data Tool"* routine is only up to third degree. So the students exported the variables of the data of their interest (in this case, *t* and *x*), in order to apply the method of least squares, as per the Numerical Analysis Program, to determine the polynomial that best adapts to the data -which was fourth degree. Table 8 shows some of the elements that the student calculated using the MathCad program.

Table 7: ***Data exported to adjust the polynomial using MathCad.***

t	0	.167	.333	.5	.667	.833	1	1.17	1.33	1.5	1.67	1.83	2	2.17
x	0	.076	.446	.993	1.48	2.2	3	3.9	4.88	5.96	6.99	8.16	9.31	1.04

Table 8. ***Mathematical elements calaculted using MathCad software.***

$f(t) = -.0006t^4 - 0.4938t^3 + 3.0937t^2 + 0.4399t - 0.0225$	Polynomial that adjusts to the runner's data



8.4 Survey analysis

The survey was made up of six blocks that related to aspects of the problem solving process, the use of the Tracker program, the teacher's role, carrying out activities, group performance, individual performance, the activities, the group presentation and three open questions.

Based on the answers provided by the students, one can sense that they possess mathematical problem solving abilities; they learn better if they work collaboratively; they present ideas; they listen to their peers, come to agreements, hang out with their peers and learning is encouraged. They enjoyed working with the Tracker program; think it useful, and consider that its use facilitates understanding of real situations such as mathematics. In addition use of the software increases the interest and motivation to learn mathematics. They also felt that their teacher's attitude and willingness to foster learning is important.

Question: Do you think use of technology is important?

All answers agreed that technology is important for learning mathematics. The following is the most representative answer:

Answer: *"Yes, I think so, due to the fact that people are always looking for a way to use the necessary resources in such a way that it makes work easier. At this time, in which technology is revolutionizing the world it is important to use it more for learning and less for leisure."*

Question: Do you think the use of real situations is important in mathematics? Justify your answer.

Answers given agree that the use of real situations in mathematics enables the understanding of concepts and mathematical algorithms, and the relating of every day situations with mathematics. Some interesting answers are shown below.

Answer: *"Yes, because that demonstrates that mathematics is indeed necessary and not only some useless knowledge. It also motivates people to awaken an enjoyment for mathematics."*

Question: In reference to student comments:

Comment 1: *"If this program (the Tracker software) were used to teach mechanics, the situation would surely be different for students."*

Comment 2: *"It's fantastic that a classroom that began from scratch with a few classmates who barely knew each other established a framework for friendship due to these kinds of activities, but especially how we all know this software (the Tracker Software) and the experience we got from working with it."*

Comment 3: *"I loved the way we worked in this course. I felt really comfortable and think that I can better explore my abilities because of it. I liked the way we worked on the project, not too overwhelming or tedious like other things. I like the way the teacher gives her class, in an interesting and fun way."*

Comment 4: *"I suggest more of these kinds of teaching methods are employed for other subjects and if possible with all topics. I also suggest all students be encouraged to bring their own laptops."*

Comment 5: *"It's a good idea to have programs like Tracker to complement a Numerical Analysis class. It should be mandatory for this course to allow the application of programs like MathCad and Tracker. Perhaps more time to familiarize ourselves with the software could have given us better results when doing the project."*

Comment 6: *"As an anecdotal piece of information, I was not expecting this to be my star course for the semester. I was wrong. I ended up loving the course, the applications, the methods, etc. All this added to the fact that the teacher really focuses on teaching. Congratulations on developing this course and way of teaching mathematics"*

Comment 7: *"I think the modeling activity was very good, because not every course we take allows us to use what we learn."*

Comment 8: *"I think the group work is good because it encourages a commitment to studying. It is also conducive to greater trust and therefore to more doubts being clarified."*

8. Conclusions

With the use of technology and collaborative work, activities that involve problem situations in mathematics courses allow students to play an active role in their own learning. Through their own experience, they build concepts and develop abilities. Through peer interaction, the ideas, knowledge, arguments and conclusions presented by each member of the group so as to defend their positions, their conceptions and points of view based on prior experience, knowledge and the skills acquired throughout their education are all strengthened and enriched.

The choice of problem situations that relate to the student's context generates certain advantages over the algorithmic activities usually worked on in a classroom setting. It puts student face to face with a completely different and real scenario that motivates them to actively learn mathematics while having fun and a clear objective given that students find the concepts and mathematics algorithms learned in the classroom useful.

Mathematical modeling as a didactic resource in and outside of the classroom, applied to each dally life situation that was assigned to each collaborative team is a good option for students to discuss and express their ideas, perspectives and personal knowledge. It fosters interest in learning mathematics through its connection to situations in context, in addition to enabling students to actively and dynamically engage in their learning processes.

The use of ICTs -the Tracker software and digital video in particular- is primarily the teacher's responsibility. The teacher must be mindful of the importance of these tools as well as of the time that students must invest in learning how to use them. In addition the teacher should be mindful of prioritizing activities to be carried out, that is activities that reflect semiotic representations, the treatment of and conversions between records. Teachers also need to recall that use of ICT should be seen as advantageous because students find them interesting and motivating.

The use of technology motivates students to learn mathematics. It facilitates interpretation of data and graphs obtained from real and easily identifiable situations. It also enables students to build knowledge, reflect on the procedures that were used, the parameters and variables that intervened in the analysis of the phenomenon in question. Collaborative work is an important feature of mathematical modeling, and complemented by problem resolution, it makes the learning of mathematics interesting for students, in particular adjusting real polynomials of one real variable. +

The digital video used in the problem situations provides an easy and efficient way of obtaining graphs and numerical data, which were then interpreted by the students to get the best representation of the phenomena. Lastly, it is paramount to highlight the importance of letting students know that teachers are not only interested in seeing them pass an exam, but they also want them to learn. Students need to feel important and supported by their teacher. This was clear in the emphasis they placed on expressing their appreciation for the way their teacher treated them and for the activities the teacher designed in order to enhance their learning experience.

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